CALCULATION OF HEAT TRANSFER THROUGH A POROUS MULTILAYER VACUUM INSULATION

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A system of differential equations describing heat and mass transfer under steady-state conditions in porous vacuum insulations is obtained and the method of their solution is discussed. The effect of various factors on heat transfer by gases in such porous media is shown.

An analysis of the mechanism of heat transfer in multilayer vacuum insulation consisting of aluminum foil and glass interlayers showed that, with free laying of the insulation, the transfer of heat by residual gas molecules located between the layers of insulation constitutes an appreciable portion (up to 70-80%) of the total heat flux even at a pressure below $1 \cdot 10^{-3}$-$1 \cdot 10^{-4}$ N/m^2 in the space surrounding the insulation [1, 2]. The presence of residual gas in the layers of insulation is due to the dynamic equilibrium of the processes of evacuation and of gas liberation from the surfaces of the insulating materials.

At a pressure around the insulation of less than $1 \cdot 10^{-2}$ N/m^2 the pressure inside it is usually no higher than 1 N/m^2 [1, 2, 8]. Since the distance between individual surfaces in such insulation is less than 0.5 mm, the flow of gas through porous insulation is molecular in nature. If follows from the kinetic gas theory that, for a molecular mechanism, the heat transfer by a gas between two adjacent surfaces is proportional to its pressure [3]:

$$q_g = \frac{1}{2} a_0 \frac{\gamma+1}{\gamma-1} \left(\frac{R}{2\pi MT} \right)^{1/2} \Delta T_i p(x). \tag{1}$$

The gas pressure in the insulation layers can be reduced both by decreasing the evolution of gas from the insulating materials and by improving the conditions of their evacuation by constructional methods. The latter includes perforation of the reflecting screens or their manufacture in the form of porous structures, for example, from metallic or metallized fibers [4, 5]. It is completely obvious that the packing material between the screens should also have good permeability for gas molecules. The insulation produced by such methods can be regarded as a porous structure permeable for gas in a direction perpendicular to the surface of the screens.

Insulation with perforated screens can be regarded as a porous medium only when the distances between the holes are sufficiently small (at least commensurable with the distances between screens) so that its gas permeability is comparable to the gas permeability of the fibrous packing material. It should be noted, however, that the construction of the reflecting screens in this case should be such that the transmission of radiation by them is minimum.

It is known that in the insulations being considered three modes of heat transfer – radiation, conduction by the gas, and conduction by the solid – act simultaneously, the fraction of each varying from screen to screen. In the absence of a heat supply to the ends of the multilayer vacuum insulation, the reflecting screens (as a consequence of the large conductivity in a longitudinal direction) can be regarded as isothermal surfaces. Therefore, determination of heat transfer is such insulations reduces to the one-dimensional problem.

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The total heat transfer $\lambda(T, x)$ can be considered to be composed additively of the contributions from the radiation $\lambda_{rad}(T)$, from the residual gas $\lambda_{g.ef}(T, x)$, and from the solid $\lambda_s(T, x)$

$$\lambda(T, x) = \lambda_{rad}(T) + \lambda_{g.ef}(T, x) + \lambda_s(T, x) \tag{2}$$

only in the space between two adjacent screens and providing that they are separated by an "optically transparent" medium. By an "optically transparent" medium we mean a material which for all practical purposes does not change the magnitude of the radiative heat transfer between adjacent screens. Such materials can include, for example, glass-fiber interlayers of the types SBR-R and EVTI [5]. In the case of small thicknesses the fiber interlayers of the indicated types will also have good gas permeability in a transverse direction owing to their large porosity (m > 0.9).

Taking into account the above conditions, we will determine the components of Eq. (2) on the basis of the relations

$$\lambda_{rad} = \frac{q_{rad}\delta}{\Delta T_i N}, \quad \lambda_{g.ef} = \frac{q_g \delta}{\Delta T_i N}.$$

For conduction by radiation, using the Stefan–Boltzmann equation, we obtain the relation

$$\lambda_{rad}(T) = 4\sigma\varepsilon_{re}(T)T^3\frac{\delta}{N}. \tag{3}$$

Heat conduction by residual gases with consideration of Eq. (1) can be written in the form

$$\lambda_{g.ef}(T, x) = \frac{1}{2}a_0\frac{\gamma+1}{\gamma-1}\left(\frac{R}{2\pi MT}\right)^{1/2} p(x)\frac{\delta}{N}. \tag{4}$$

Conduction by the solid is determined mainly by the contact resistance of the packing material and in the case of free laying of insulation its contribution to total heat transfer is small (to 5-10%) [2] and can be neglected in the first approximation.

As we see from Eqs. (3) and (4), conduction by radiation and residual gases decreases with a decrease of the characteristic dimension δ/N between adjacent surfaces. From this viewpoint it is desirable to use fine-fiber interlayers of minimum thickness. However, the reduction in their thickness is limited by the need to provide sufficient thermal resistance of the material of the interlayers between the adjacent screens.

It should be noted that the conductions obtained by Eqs. (3) and (4) have constant values in the space between adjacent screens. This is sufficiently valid in view of the small thickness of the interlayers. Nevertheless, taking into account the high density of screens and their large number, we can consider the functions describing the conductivity of the insulation and the temperature throughout its thickness as continuous and differentiable.

Heat transfer through a porous system in the presence of mass transfer is described by the energy equation, which in the general case for the one-dimensional steady-state problem can be written in the form [9, 10]

$$\rho v c_p \frac{dT}{dx} - v\frac{dp}{dx} = \frac{d}{dx}\left(\lambda\frac{dT}{dx}\right) + r\frac{d(\rho v)}{dx}. \tag{5}$$

In writing Eq. (5) the terms taking into account energy dissipation due to viscosity are eliminated, since the gas flow occurs in a molecular regime. It is easy to show that in the case in question all the remaining terms of Eq. (5) are sufficiently small in comparison with $d(\lambda dT/dx)/dx$. We will estimate the orders of magnitude of all the terms of Eq. (5). To estimate the maximum possible value of the left-hand side of the equation, we will consider the gas pressure in the layers of insulation to be constant and equal to its maximum value, and the product ρv to be equal to its maximum value on the evacuated side of the insulation $(\rho v)_{max} = W_0(T_1)\delta$. In this case the following relations will be valid:

$$\rho v c_p\frac{dT}{dx} - v\frac{dp}{dx} < c_p \Delta T W_0(T_1),$$

$$r\frac{d(\rho v)}{dx} < rW_0(T_1),$$

$$\frac{d}{dx}\left(\lambda\frac{dT}{dx}\right) \approx \frac{\lambda_{ef}\Delta T}{\delta^2}.$$

An evaluation of these quantities shows that even for the most effective insulations with a thickness to 10 cm, the heat transfer related with mass transfer and internal heat sources (heat of desorption of the gases being evacuated) does not exceed 1% of the conductive heat transfer and can be neglected. Thus, for calculating heat transfer through porous multilayer vacuum insulation under steady-state conditions we obtain the equation

$$\frac{d}{dx}\left\{\left[4\sigma\varepsilon_{\mathrm{re}}(T)T^3\frac{\delta}{N}+\frac{1}{2}a_0\frac{\gamma+1}{\gamma-1}\left(\frac{R}{2\pi MT}\right)^{\frac{1}{2}}p(x)\frac{\delta}{N}\right]\frac{dT}{dx}\right\}=0.$$

Providing the density of insulation layers is constant over the entire thickness, the characteristic dimension δ/N = const and the preceding equation take the form

$$\frac{dT}{dx}\left[4\sigma\varepsilon_{\mathrm{re}}(T)\,T^3+\frac{1}{2}a_0\frac{\gamma+1}{\gamma-1}\left(\frac{R}{2\pi MT}\right)^{\frac{1}{2}}p(x)\right]=\text{const.} \tag{6}$$

The boundary conditions for Eq. (6) are the following:

$$T(0)=T_1,\quad T(\delta)=T_2. \tag{7}$$

As we see from (6), the total heat transfer $\lambda(T, x)$ depends on the distribution of pressure over the thickness of the insulation $p(x)$. In turn the pressure distribution is affected by the temperature of the screens, since the gas evolution of materials is a function of temperature. Thus, Eq. (6) must be solved simultaneously with the mass-transfer equation, which also describes the relation between functions $p(x)$ and $T(x)$ (see (11)).

One of the possible methods of solution is the method of successive approximation; it is adequate to take a linear distribution of temperature between T_1 and T_2 as the first approximation for finding $p = p(x)$ from Eq. (11). The value of $p(x)$ obtained is substituted into (6) for refining the functional relation $T(x)$, and so on. We can obtain this solution in practice only with the help of numerical methods, knowing the temperature dependences of the emissivity of the reflecting screens $\varepsilon(T)$ and the accommodation coefficients $a(T)$ [1, 5].

We will consider the differential equation of mass transfer corresponding to the conditions taking place in porous multilayer vacuum insulations.

For a molecular regime, the quantity of gas passing in unit time through unit surface in the direction of evacuation is determined according to Deryagin's theory [6] by the equation

$$Q=-\frac{k}{\sqrt{T}}\,\frac{dp(x)}{dx}, \tag{8}$$

where

$$k=\frac{24}{13}\sqrt{\frac{2}{\pi MR}}\,\frac{m^2}{S_0}.$$

This expression holds in the absence of internal sources and sinks of the gas mass. In the insulation being considered an evacuation flow is formed after removal of the original gas mass and the arrival at a steady regime owing to gas evolution from the surface of the materials. Therefore, in Eq. (8) it is necessary to take into account internal gas evolution, which for a unit volume of insulation can be represented in the form

$$W_0(T)=A_s(T)S_{0s}+A_i(T)+S_{vi}\ . \tag{9}$$

Mass transfer due to effusion occurring under a molecular regime in the capillaries, as a consequence of a temperature difference, can be neglected since this flow ($Q_s = (8l/3)(M/2\pi R)^{1/2}\,d(p/\sqrt{T})/dx$) [10] is smaller by two orders than the filtration transfer described by Deryagin's formula.

On the basis of the law of conservation of mass for a gas passing through a volume element (Fig. 1), we obtain the relation

$$Q(x)+W_0(T)\,dx=Q(x)+\frac{dQ(x)}{dx}dx. \tag{10}$$

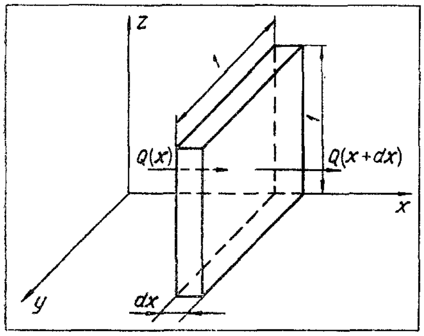

Fig. 1. Gas flow through a volume element of insulation.

Thus, taking into account Eq. (8), the equation of mass transfer in porous multilayer vacuum insulations acquires the form

$$\frac{d^2p(x)}{dx^2}-\frac{1}{2T(x)}\cdot\frac{dT(x)}{dx}\cdot\frac{dp(x)}{dx}+\frac{\sqrt{T(x)}}{k}W_0(T)=0. \tag{11}$$

The problem of determining heat transfer in porous multilayer vacuum insulations, as was indicated above, reduces to the solution of the system of differential equations (6) and (11).

A determination of the extent to which various factors affect the magnitude and character of the pressure distribution in the insulation layers is of great practical interest. To estimate the maximum possible pressure we will consider the evacuation of insulation in a thermal state. Since in this case the temperature over the thickness of the insulation is constant, Eq. (11) acquires the form

$$\frac{d^2p(x)}{dx^2}+\frac{\sqrt{T_1}}{k}W_0(T_1)=0. \tag{12}$$

It has the following solution:

$$p(x)=-\frac{\sqrt{T_1}}{k}W_0(T_1)\frac{x^2}{2}+c_1x+c_2. \tag{13}$$

To find the boundary conditions we will consider the most typical case, when evacuation of the insulation occurs on one side. If coordinate x is reckoned from the surface of the insulation on the unevacuated side, we obtain

$$p(\delta)=p_0,\quad \left.\frac{dp(x)}{dx}\right|_{x=0}=0. \tag{14}$$

The last relation reflects the absence of a gas flow on the unevacuated side of the insulation.

Using conditions (14), we represent Eq. (13) in the form

$$p(x)=p_0+\frac{13}{48\sqrt{2}}\sqrt{\pi MRT}\,\frac{S_0}{m^2}W_0(T_1)(\delta^2-x^2). \tag{15}$$

In the case of constant temperature over the thickness of the insulation, the pressure in the layers changes according to a parabolic law with a maximum at $x = 0$, i.e., on the unevacuated side of the insulation. The total pressure drop Δp through the insulation is

$$\Delta p=p(0)-p_0=\frac{13\sqrt{\pi MRT}}{48\sqrt{2}}\,\frac{S_0W_0(T_1)}{m^2}\delta^2. \tag{16}$$

Thus, the pressure in the layers of insulation is determined not only by gas liberation $W_0(T)$ of the materials of the insulation but also by its structure (specific surface S_0 and porosity m), dimensions (thickness δ), and composition of the residual gas (molecular weight M).

Actually, Eq. (16) determines the minimum attainable pressure on the unevacuated side of the insulation. In this case, as we see from (15) and (16), no decrease of pressure p_0 whatever can effect a further decrease of the indicated pressure $p(0)$, which will be determined by the relation between the gas evolution and gas permeability of the insulation structure. Equation (16) also shows that, along with the creation of porous insulations, the use of materials with low gas evolution is an effective method of improving multilayer vacuum insulation.

The specific surface S_0 in Eq. (16) is the sum of the specific surfaces of the screens and packing material. The specific surface of flat screens is easily calculated owing to the simplicity of their geometry. The specific surface of fibrous structures can be found only experimentally [7] or by calculation if the fibers have about the same thickness. In this case the specific surface of fibrous material is

$$S_i=\frac{4(1-m)}{d}. \tag{17}$$

The magnitude of the specific surface of fibrous interlayers per unit volume of insulation is determined by the expression

$$S_{vi} = \frac{4(1-m)\delta_i N}{d\delta}. \tag{18}$$

Calculations carried out for insulation 10 cm thick consisting of fibrous interlayers and screens of the same structure made from metallized fibers with a laying density of 20 screens/cm gave a value of $1 \cdot 10^{-3}$ N/m^2 for the pressure drop through the insulation. It was assumed that $T_1 = 300°K$, $m = 0.93$; $A_s = A_i = 1 \cdot 10^{-7}$ liter·torr/cm²·sec; $d = 5\ \mu$; $\delta_i = 20\ \mu$.

In real insulation the pressure distribution will have a more complex character owing to the presence of a temperature gradient. In this case, to calculate its effectiveness it is necessary to have data on the temperature and time dependence of the gas evolution of various materials, the composition of the gases being evolved, the temperature dependence of emissivity and accommodation, etc. At present such data are practically absent and considerable experimental work is necessary to determine them.

NOTATION

q_{rad}, q_g	are the specific heat fluxes transferred by radiation and residual gases;
a_0	is the reduced accommodation coefficient;
$\gamma = c_p/c_v$	is the ratio of the specific heats at constant pressure and constant volume;
R	is the universal gas constant;
M	is the molecular weight;
T, T_1, T_2	are the current temperature and the temperature of the warm and cold walls of the insulation, respectively;
$\Delta T = T_1 - T_2$	is the temperature drop over the thickness of the insulation;
ΔT_i	is the temperature difference in adjacent screens;
p	is the gas pressure;
x	is the moving coordinate over the insulation thickness;
λ, λ_{rad}, $\lambda_{g.ef}$, λ_s	are the coefficient of total heat conduction and conductions by radiation, gas, and solid in each section of the insulation, respectively;
λ_{ef}	is the effective thermal conductivity of the insulation;
δ, N	are the thickness and number of insulating screens, respectively;
σ	is the Stefan–Boltzmann constant;
ε_{re}	is the reduced emissivity of two adjacent surfaces;
Q	is the mass flow of gas through unit surface in unit time;
m	is the average volume porosity of the material;
S_0	is the specific surface, i.e., the area of all surfaces per unit volume;
W_0	is the gas evolution per unit volume of insulation in unit time;
A	is the mass rate of gas evolution per unit surface;
d	is the diameter of fibers;
δ_i	is the thickness of interlayers;
ρ	is the gas density;
v	is the gas velocity;
r	is the heat of desorption of gas;
l	is the average size of pores;
p_0	is the pressure maintained on evacuated side of insulation.

Subscripts

s	is the screen;
i	is the interlayer.

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